CHAPTER FIVE

DENSITY, ARCHEMEDES PRINCIPLE AND FLOATING BODIES

Density:

- The density of a sibstance is defined as its mass per unit volume.

- Density = $\frac{Mass}{Volume}$ or $e = \frac{m}{v}$,

where e = density, m = mass and v = volume.

Importance of density measurement:

(1) It can be used to determine whether a substance or an item is pure or not.
(2) Architects and engineers make us of the densities of materials, when they design structures such as bridges and flyovers.

Simple measurement of density:

(1) Liquid:

- To measure the density of a liquid, a convenient volume of the liquid is run off into a clean, dry, previously weighed beaker, using eihter a pipettte or burette.
- The beaker and the liquid are then weighed and the mass of the liquid found by subtraction.
- The volume of the liquid delivered by the pipette or the burette is noted.
- We finally divide the mass by the volume to get the density.

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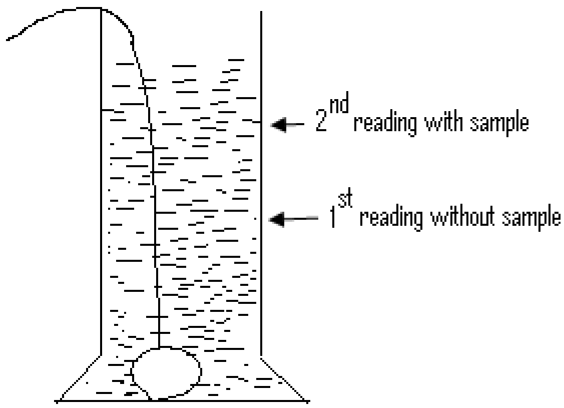
(2) Solid:



- If the substance has a regular shape e.g cylinder, sphere or a rectangular bar, its volume can be found by calculation.

- Its mass can then be found through weighing.-
- The mass is then divided by the volume to get the density.
- If the solid has an irregular shape, such as that of a stone, then its vol;ume is found in the following manner.
- Water is first placed inside a measuring cylinder and its volume is noted.
- The stone tied to a thread is completely immersed into the water in the cylinder.- The new volume of the water is also noted, and the difference between the two volumes gives us the volume of the solid.
- The mass of the solid is found through weighing and we divide the mass by the volume to get the density.

Relative density:

- The relative density of a substance is the ratio of the mass of any volume of it, to the mass of an equal volume of water.

- Relative density = $\frac{mass\ of\ any\ volume\ of\ the\ substances}{mass\ of\ equal\ volume\ of\ water}$

- Because the mass of a body is proportional to its weight

=> relative density = $\frac{weight\ of\ any\ volume\ of\ the\ substances}{weight\ of\ an\ equal\ volume\ of\ water}$

- This explains why relative density is also called specific gravty, since gravity implies weight.
- Relative density has no units but density is expressed in kg/m^3 or g/cm^3.

Measuring the relative density of a liquid:

- To ensure getting identical equal volumes of water and a liquid, we make use of the density bottle.
- This botttle is first weighed empty, and later weighed when it is filled with the given liquid.
- The liquid is then removed from the density bottle, and after rinsing or cleaning the bottle, it is filled with water and weighed again.
- To get the mass of the liquid, the mass of the empty bottle is subtracted from the mass of the bottle filled with the liquid.- To get the mass of the water, we subtract the mass of the empty bottle from the mass of the bottle filled with water.

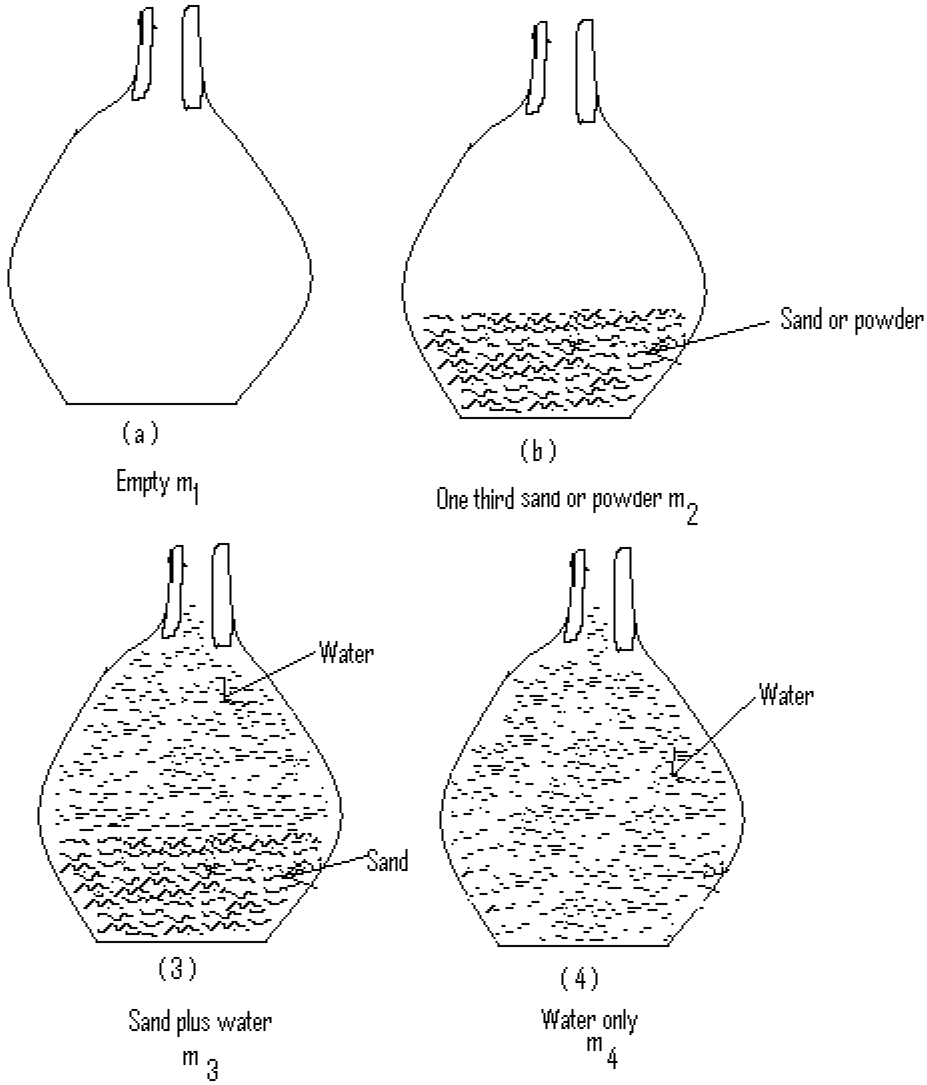
- To get the relative density, we divide the mass of the liquid by the mass of the water.
- The precautions taken are:

  (I) The outside of the bottle should be wiped dry before weighing.

  (I) The bottle should not be held in warm hands, otherwise some of the liquid may be lost through expansion.

Measuring the relative density of a powder:

- In this case, the powder must not be soluble in water, and the same method can be used to determine the relative density of other items which are insoluble in water, and are in the powdered or granular form such as sand.
- The density bottle is used in this case.



Relative density of sand (powder) = $\frac{mass\ of\ any\ volume\ of\ sand\ (powder)}{mass\ of\ an\ equal\ volume\ of\ water}$

$$= \frac{mass\ of\ sand\ in\ (2)}{mass\ of\ water\ in\ (4) - mass\ of\ water\ in (3)}$$

$$= \frac{m_2 - m_1}{(m_4 - m_1) - (m_3 - m_2)}$$

(Q1) A piece of a material has a volume of 15cm^3 and a mass of 27g. Calculate its density in g/cm^3.

Soln:

Mass = m = 27g

Volume = $15cm^3$

Density $e = \frac{m}{v} = \frac{27}{15}$

= $1.8gcm^{-3}$

N/B: The density of water = $1g/cm^3$.

(Q2) Calculate the mass of air in a room of floor dimension 10m $\times 12m$ and height 4m. (Density of air = $1.26kg/m^3$).

Soln:

Volume of the room = L $\times$ B $\times$ H = 12 $\times$ 10 $\times$ 4 = $480m^3$.

Mass = ?

Density = (e) = $\frac{mass}{volume}$

=> mass = density $\times$ volume

=> mass = 1.26 $\times$ 480 = 605kg.

(Q3) An empty 60litre petrol tank weighs 10kgf. Determine its weight when it is filled with a fuel of relative density 0.72.

Soln:

Volume = 60 litres.

Relative density = 0.72.

Relative density = $\frac{mass}{volume}$

=> 0.72 = $\frac{mass}{volume}$

=> 0.72 = $\frac{mass}{60}$

=> mass = 0.72 $\times$ 60 = 43.2kg,

=> the mass of fuel = 43.2kg.

But since the mass of the tank is 10kgf => the mass of the tank when filled with fuel = 43.2 + 10 = 53.2kg.

(Q4) A bottle full of water weighs 45gf. When it is full of mercury, it weighs 360gf. If the empty bottle weighs 20.0gf, calculate the density of mercury.

Soln:

Weight of bottle filled with water = 45gf.

Weight of empty bottle = 20gf,

=> weight of water = 45 – 20 = 25g (i.e. weight of water within the density bottle).

Density = $\frac{mass}{volume}$

Since the density of water = 1g/cm^3 => 1 = $\frac{25}{v}$

=> V = $\frac{25}{1} = 25cm^3$

=> the volume of the bottle = 25cm^3.

Mass of bottle filled with mercury = 360gf.

Since the mass of the bottle = 20gf => the mass of mercury = 360 – 20 = 340gf.

Volume of the bottle = 25cm^3

Density = $\frac{mass}{volume}$ => density of mercury = $\frac{340}{25}$ = 13.6gcm^{-3}.

(Q5) A density bottle weighs 18gf when empty, 44gf when full of water and 40gf when full of a second liquid. Determine the density of this liquid.

Soln:

Weight of empty bottle = 18gf.

Weight of bottle filled with water = 44gf,

=> weight of water = 44 – 18 = 26g.

But density = $\frac{mass}{volume}$, and since the density of water = 1g/ cm^3

=> 1 = $\frac{26}{v} => v =$26cm^2 => the volume of the bottle = 26cm^3.

Since weight of the bottle filled with the second liquid = 40g, => weight of the second liquid = 40 – 18 = 22g.

Volume of bottle = 26cm^3

Density of this liquid $= \frac{mass}{volume} = \frac{22}{26}$

$= 0.9gcm^{-3}$.

Archimedes principle and floating bodies:

Upthrust: When a body is placed in a liquid, an upward force (i.e. a force which acts in the upward direction) acts on it.This upward acting force is called the upthrust.

Apparent loss in weight:

- A simple but striking experiment to illustrate the upthrust exerted by a liquid, can be shown by tying a length of cotton to a brick.
- Any attempt made to lift the brick by the cotton fails through the breakage of the cotton.
- But if the brick is immersed in water, it may be lifted quite easily.
- This is due to the fact that because of the upthrust exerted on the brick by the water, it appears to weigh less in water, than in air.
- In short, there is a decrease in the weight of a body when it is placed in a liquid such as water.

- Stone boulders immersed in water have an upthrust acting on them, which causes decreases in their weights.
- This explains why these boulders can so easily be moved by flood waters.

Archimedes principle:

Experiment to measure the upthrust of a liquid was first carried out by Archimedes, who came out with a discovery called the Archimedes principle. This principle states that when a body is wholly or partially immersed in a fluid, it experiences an upthrust which is equal to the weight of the fluid displaced by that body. The word fluid may either be referring to either a liquid or a gas.

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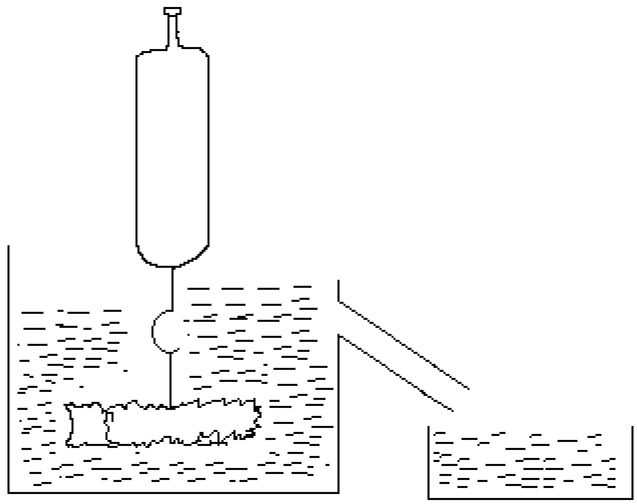
To verify Archimede principle for a body in liquid:



- An eureka or displacement can is placed on a bench, with a beaker under its spout.
- Water is then poured into the can until it rans from the spout.
- When the water has ceased dropping, the beaker is removed and replaced by another which has previously been dried and weighed.
- Any suitable solid body such as a piece of metal or stone, is suspended by a thread from the hook of a spring – balance, and the weight of the body in air is measured.
- The body, still attached to the balance is carefully lowered into the displacement can, and when it is completely immersed, its new weight is noted.

- The displaced water is caught in a weighed beaker, and when no more water drips from the spout, the beaker and the water is weighed.
- The result should be set down as follow:
  Weight of body in air =
  Weight of body in water =
  Weight of empty beaker =
  Weight of water plus displaced water =

- Apparent loss in weight of body = the weight of the water displaced.
- The apparent loss of weight of the body, or the upthrust on it, should be equal to the weight of water displaced.
- This verifies Archimedes principle in the case of water.
- A similar result will be obtained, if any other liquid is used.

To measure the relative density of a solid by using Archimedes principle:

- The relative density of a substance = $\frac{mass\ of\ any\ volume\ of\ the\ substance}{mass\ of\ an\ equal\ volume\ of\ water}$

- Archimedes principle gives us a simple but accurate method for finding the relative density of a solid.
- If we take a sample of the solid and weigh it first in air, and then in water, the apparent loss in weight obtained by subtraction, is the volume of water displaced by that sample.

- The relative density of a substance = $\frac{weight\ of\ a\ sample\ of\ substance}{apparent\ loss\ in\ weight\ of\ sample\ in\ H_2O}$

- For example, this method can be used to determine the relative density of a piece of brass.
- The weight of the brass is first determined in air.-
- The brass is then completely immersed in a beaker which contains water.
- The results are recorded as follows:

Weight of brass in air = …………………. gf.
Weight of water = ………………

Apparent loss in weight of brass =

Relative density of brass = $\frac{Weight\ of\ brass\ in\ air}{apparent\ loss\ in\ weight\ in\ H_2O}$